

# Package ‘FKF’

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**Description** This is a fast and flexible implementation of the Kalman filter, which can deal with NAs. It is entirely written in C and relies fully on linear algebra subroutines contained in BLAS and LAPACK. Due to the speed of the filter, the fitting of high-dimensional linear state space models to large datasets becomes possible. This package also contains a plot function for the visualization of the state vector and graphical diagnostics of the residuals.

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fkf

*Fast Kalman filter***Description**

This function allows for fast and flexible Kalman filtering. Both, the measurement and transition equation may be multivariate and parameters are allowed to be time-varying. In addition “NA”-values in the observations are supported. `fkf` wraps the C-function `FKF` which fully relies on linear algebra subroutines contained in `BLAS` and `LAPACK`.

**Usage**

```
fkf(a0, P0, dt, ct, Tt, Zt, HHt, GGt, yt, check.input = TRUE)
```

**Arguments**

<code>a0</code>	A vector giving the initial value/estimation of the state variable.
<code>P0</code>	A matrix giving the variance of <code>a0</code> .
<code>dt</code>	A matrix giving the intercept of the transition equation (see <b>Details</b> ).
<code>ct</code>	A matrix giving the intercept of the measurement equation (see <b>Details</b> ).
<code>Tt</code>	An array giving the factor of the transition equation (see <b>Details</b> ).
<code>Zt</code>	An array giving the factor of the measurement equation (see <b>Details</b> ).
<code>HHt</code>	An array giving the variance of the innovations of the transition equation (see <b>Details</b> ).
<code>GGt</code>	An array giving the variance of the disturbances of the measurement equation (see <b>Details</b> ).
<code>yt</code>	A matrix containing the observations. “NA”-values are allowed (see <b>Details</b> ).
<code>check.input</code>	A logical stating whether the input shall be checked for consistency (“storage.mode”, “class”, and dimensionality, see <b>Details</b> ).

**Details****State space form:**

The following notation is closest to the one of Koopman et al. The state space model is represented by the transition equation and the measurement equation. Let  $m$  be the dimension of the state variable,  $d$  be the dimension of the observations, and  $n$  the number of observations. The transition equation and the measurement equation are given by

$$\alpha_{t+1} = d_t + T_t \cdot \alpha_t + H_t \cdot \eta_t$$

$$y_t = c_t + Z_t \cdot \alpha_t + G_t \cdot \epsilon_t,$$

where  $\eta_t$  and  $\epsilon_t$  are iid  $N(0, I_m)$  and iid  $N(0, I_d)$ , respectively, and  $\alpha_t$  denotes the state variable. The parameters admit the following dimensions:

$$\begin{array}{lll}
a_t \in R^m & d_t \in R^m & \eta_t \in R^m \\
T_t \in R^{m \times m} & H_t \in R^{m \times m} & \\
y_t \in R^d & c_t \in R^d & \epsilon_t \in R^d. \\
Z_t \in R^{d \times m} & G_t \in R^{d \times d} & 
\end{array}$$

Note that `fkf` takes as input `HHt` and `GGt` which corresponds to  $H_t H_t'$  and  $G_t G_t'$ .

### Iteration:

Let `i` be the loop variable. The filter iterations are implemented the following way (in case of no NA's):

Initialization:

```

if(i == 1){
at[, i] = a0
Pt[, , i] = P0
}

```

Updating equations:

```

vt[, i] = yt[, i] - ct[, i] - Zt[, , i] %% at[, i]
Ft[, , i] = Zt[, , i] %% Pt[, , i] %% t(Zt[, , i]) + GGt[, , i]
Kt[, , i] = Pt[, , i] %% t(Zt[, , i]) %% solve(Ft[, , i])
att[, i] = at[, i] + Kt[, , i] %% vt[, i]
Ptt[, i] = Pt[, , i] - Pt[, , i] %% t(Zt[, , i]) %% t(Kt[, , i])

```

Prediction equations:

```

at[, i + 1] = dt[, i] + Tt[, , i] %% att[, i]
Pt[, , i + 1] = Tt[, , i] %% Ptt[, , i] %% t(Tt[, , i]) + HHt[, , i]

```

Next iteration:

```

i <- i + 1
goto "Updating equations".

```

### NA-values:

NA-values in the observation matrix `yt` are supported. If particular observations `yt[, i]` contain NAs, the NA-values are removed and the measurement equation is adjusted accordingly. When the full vector `yt[, i]` is missing the Kalman filter reduces to a prediction step.

### Parameters:

The parameters can either be constant or deterministic time-varying. Assume the number of observations is  $n$  (i.e.  $y = (y_t)_{t=1, \dots, n}$ ,  $y_t = (y_{t1}, \dots, y_{td})$ ). Then, the parameters admit the following classes and dimensions:

<code>dt</code>	either a $m \times n$ (time-varying) or a $m \times 1$ (constant) matrix.
<code>Tt</code>	either a $m \times m \times n$ or a $m \times m \times 1$ array.
<code>HHt</code>	either a $m \times m \times n$ or a $m \times m \times 1$ array.
<code>ct</code>	either a $d \times n$ or a $d \times 1$ matrix.
<code>Zt</code>	either a $d \times m \times n$ or a $d \times m \times 1$ array.
<code>GGt</code>	either a $d \times d \times n$ or a $d \times d \times 1$ array.
<code>yt</code>	a $d \times n$ matrix.

If `check.input` is TRUE each argument will be checked for correctness of the dimensionality, storage mode, and class. `check.input` should always be TRUE unless the performance becomes crucial

and correctness of the arguments concerning dimensions, class, and storage.mode is ensured. Note that the class of the arguments is of importance. For instance, to check whether a parameter is constant the `dim` attribute is accessed. If, e.g., `Zt` is a constant, it could be a  $d \times d$ -matrix. But the third dimension (i.e. `dim(Zt)[3]`) is needed to check for constancy. This requires `Zt` to be an  $d \times d \times 1$ -array.

#### BLAS and LAPACK routines used:

The R function `fkf` basically wraps the C-function `FKF`, which entirely relies on linear algebra subroutines provided by BLAS and LAPACK. The following functions are used:

BLAS: `dcopy`, `dgemm`, `daxpy`.  
LAPACK: `dpotri`, `dpotrf`.

`FKF` is called through the `.Call` interface. Internally, `FKF` extracts the dimensions, allocates memory, and initializes the R-objects to be returned. `FKF` subsequently calls `cfkf` which performs the Kalman filtering.

The only critical part is to compute the inverse of  $F_t$  and the determinant of  $F_t$ . If the inverse can not be computed, the filter stops and returns the corresponding message in `status` (see **Value**). If the computation of the determinant fails, the filter will continue, but the log-likelihood (element `logLik`) will be “NA”.

The inverse is computed in two steps: First, the Cholesky factorization of  $F_t$  is calculated by `dpotrf`. Second, `dpotri` calculates the inverse based on the output of `dpotrf`.

The determinant of  $F_t$  is computed using again the Cholesky decomposition.

#### Value

An S3-object of class “`fkf`”, which is a list with the following elements:

- `att` A  $m \times n$ -matrix containing the filtered state variables, i.e.  $a_{t|t} = E(\alpha_t|y_t)$ .
- `at` A  $m \times (n + 1)$ -matrix containing the predicted state variables, i.e.  $a_t = E(\alpha_t|y_{t-1})$ .
- `Ptt` A  $m \times m \times n$ -array containing the variance of `att`, i.e.  $P_{t|t} = var(\alpha_t|y_t)$ .
- `Pt` A  $m \times m \times (n + 1)$ -array containing the variances of `at`, i.e.  $P_t = var(\alpha_t|y_{t-1})$ .
- `vt` A  $d \times n$ -matrix of the prediction errors given by  $v_t = y_t - c_t - Z_t a_t$ .
- `Ft` A  $d \times d \times n$ -array which contains the variances of `vt`, i.e.  $F_t = var(v_t)$ .
- `Kt` A  $m \times d \times n$ -array containing the “Kalman gain” (ambiguity, see calculation above).
- `logLik` The log-likelihood.
- `status` A vector which contains the status of LAPACK’s `dpotri` and `dpotrf`. (0, 0) means successful exit.
- `sys.time` The time elapsed as an object of class “`proc_time`”.

The first element of both `at` and `Pt` is filled with the function arguments `a0` and `P0`, and the last, i.e. the  $(n + 1)$ -th, element of `at` and `Pt` contains the predictions

$$at[, n + 1] = E(\alpha_{n+1}|y_n) \text{ and}$$

$$Pt[, , n + 1] = var(\alpha_{n+1}|y_n).$$

#### Author(s)

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## References

Harvey, Andrew C. (1990). *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge University Press.

Hamilton, James D. (1994). *Time Series Analysis*. Princeton University Press.

Koopman, S. J., Shephard, N., Doornik, J. A. (1999). *Statistical algorithms for models in state space using SsfPack 2.2*. *Econometrics Journal*, Royal Economic Society, vol. 2(1), pages 107-160.

## See Also

[plot](#) to visualize and analyze fkf-objects, [KalmanRun](#) from the stats package, function [dlmFilter](#) from package dlm, and function [kfilter](#) from package sspir.

## Examples

```
## ----->
## Example 1: ARMA(2, 1) model estimation.
## ----->
## This example shows how to fit an ARMA(2, 1) model using this Kalman
## filter implementation (see also stats' makeARIMA and KalmanRun).
n <- 1000

## Set the AR parameters
ar1 <- 0.6
ar2 <- 0.2
ma1 <- -0.2
sigma <- sqrt(0.2)

## Sample from an ARMA(2, 1) process
a <- arima.sim(model = list(ar = c(ar1, ar2), ma = ma1), n = n,
               innov = rnorm(n) * sigma)

## Create a state space representation out of the four ARMA parameters
arma2lss <- function(ar1, ar2, ma1, sigma) {
  Tt <- matrix(c(ar1, ar2, 1, 0), ncol = 2)
  Zt <- matrix(c(1, 0), ncol = 2)
  ct <- matrix(0)
  dt <- matrix(0, nrow = 2)
  GGt <- matrix(0)
  H <- matrix(c(1, ma1), nrow = 2) * sigma
  HHt <- H %*% t(H)
  a0 <- c(0, 0)
  P0 <- matrix(1e6, nrow = 2, ncol = 2)
  return(list(a0 = a0, P0 = P0, ct = ct, dt = dt, Zt = Zt, Tt = Tt, GGt = GGt,
```

```

        Hht = Hht))
    }

## The objective function passed to 'optim'
objective <- function(theta, yt) {
  sp <- arma21ss(theta["ar1"], theta["ar2"], theta["ma1"], theta["sigma"])
  ans <- fkf(a0 = sp$a0, P0 = sp$P0, dt = sp$dt, ct = sp$ct, Tt = sp$Tt,
            Zt = sp$Zt, Hht = sp$Hht, GGt = sp$GGt, yt = yt)
  return(-ans$logLik)
}

theta <- c(ar = c(0, 0), ma1 = 0, sigma = 1)
fit <- optim(theta, objective, yt = rbind(a), hessian = TRUE)
fit

## Confidence intervals
rbind(fit$par - qnorm(0.975) * sqrt(diag(solve(fit$hessian))),
      fit$par + qnorm(0.975) * sqrt(diag(solve(fit$hessian))))

## Filter the series with estimated parameter values
sp <- arma21ss(fit$par["ar1"], fit$par["ar2"], fit$par["ma1"], fit$par["sigma"])
ans <- fkf(a0 = sp$a0, P0 = sp$P0, dt = sp$dt, ct = sp$ct, Tt = sp$Tt,
          Zt = sp$Zt, Hht = sp$Hht, GGt = sp$GGt, yt = rbind(a))

## Compare the prediction with the realization
plot(ans, at.idx = 1, att.idx = NA, CI = NA)
lines(a, lty = "dotted")

## Compare the filtered series with the realization
plot(ans, at.idx = NA, att.idx = 1, CI = NA)
lines(a, lty = "dotted")

## Check whether the residuals are Gaussian
plot(ans, type = "resid.qq")

## Check for linear serial dependence through 'acf'
plot(ans, type = "acf")

## ----->
## Example 2: Local level model for the Nile's annual flow.
## ----->
## Transition equation:
##  $\alpha[t+1] = \alpha[t] + \eta[t]$ ,  $\eta[t] \sim N(0, Hht)$ 
## Measurement equation:
##  $y[t] = \alpha[t] + \epsilon[t]$ ,  $\epsilon[t] \sim N(0, GGt)$ 

y <- Nile
y[c(3, 10)] <- NA # NA values can be handled

## Set constant parameters:
dt <- ct <- matrix(0)
Zt <- Tt <- matrix(1)

```

```

a0 <- y[1]          # Estimation of the first year flow
P0 <- matrix(100)   # Variance of 'a0'

## Estimate parameters:
fit.fkf <- optim(c(HHt = var(y, na.rm = TRUE) * .5,
                  GGt = var(y, na.rm = TRUE) * .5),
               fn = function(par, ...)
                 -fkf(HHt = matrix(par[1]), GGt = matrix(par[2]), ...) $logLik,
                 yt = rbind(y), a0 = a0, P0 = P0, dt = dt, ct = ct,
                 Zt = Zt, Tt = Tt, check.input = FALSE)

## Filter Nile data with estimated parameters:
fkf.obj <- fkf(a0, P0, dt, ct, Tt, Zt, HHt = matrix(fit.fkf$par[1]),
              GGt = matrix(fit.fkf$par[2]), yt = rbind(y))

## Compare with the stats' structural time series implementation:
fit.stats <- StructTS(y, type = "level")

fit.fkf$par
fit.stats$coef

## Plot the flow data together with fitted local levels:
plot(y, main = "Nile flow")
lines(fitted(fit.stats), col = "green")
lines(ts(fkf.obj$att[1, ], start = start(y), frequency = frequency(y)), col = "blue")
legend("top", c("Nile flow data", "Local level (StructTS)", "Local level (fkf)"),
      col = c("black", "green", "blue"), lty = 1)

```

---

plot.fkf

*Plotting fkf Objects*


---

## Description

Plotting method for objects of class `fkf`. This function provides tools for graphical analysis of the Kalman filter output: Visualization of the state vector, QQ-plot of the individual residuals, QQ-plot of the Mahalanobis distance, auto- as well as crosscorrelation function of the residuals.

## Usage

```

## S3 method for class 'fkf'
plot(x, type = c("state", "resid.qq", "qqchisq", "acf"),
     CI = 0.95, at.idx = 1:nrow(x$at), att.idx = 1:nrow(x$att), ...)

```

## Arguments

`x` The output of `fkf`.

type	A string stating what shall be plotted (see <b>Details</b> ).
CI	The confidence interval in case type == "state". Set CI to NA if no confidence interval shall be plotted.
at.idx	An vector giving the indexes of the predicted state variables which shall be plotted if type == "state".
att.idx	An vector giving the indexes of the filtered state variables which shall be plotted if type == "state".
...	Arguments passed to either <a href="#">plot</a> , <a href="#">qqnorm</a> , <a href="#">qqplot</a> or <a href="#">acf</a> .

### Details

The argument type states what shall be plotted. type must partially match one of the following:

**state** The state variables are plotted. By the arguments `at.idx` and `att.idx`, the user can specify which of the predicted ( $a_t$ ) and filtered ( $a_{t|t}$ ) state variables will be drawn.

**resid.qq** Draws a QQ-plot for each residual-series `inv`.

**qqchisq** A Chi-Squared QQ-plot will be drawn to graphically test for multivariate normality of the residuals based on the Mahalanobis distance.

**acf** Creates a pairs plot with the autocorrelation function ([acf](#)) on the diagonal panels and the crosscorrelation function ([ccf](#)) of the residuals on the off-diagonal panels.

### Value

Invisibly returns an list with components:

**distance** The Mahalanobis distance of the residuals as a vector of length  $n$ .  
**std.resid** The standardized residuals as an  $d \times n$ -matrix. It should hold that  $std.resid_{ij} \text{ iid } \sim N_d(0, I)$ ,

where  $d$  denotes the dimension of the data and  $n$  the number of observations.

### Author(s)

David Luethi, Philipp Erb, Simon Otziger

### See Also

[fkf](#)

### Examples

```
## Local level model for the treering width data.
## Transition equation:
## alpha[t+1] = alpha[t] + eta[t], eta[t] ~ N(0, HHT)
## Measurement equation:
## y[t] = alpha[t] + eps[t], eps[t] ~ N(0, GGt)

y <- treering
y[c(3, 10)] <- NA # NA values can be handled
```

```
## Set constant parameters:
dt <- ct <- matrix(0)
Zt <- Tt <- matrix(1)
a0 <- y[1]          # Estimation of the first width
P0 <- matrix(100)   # Variance of 'a0'

## Estimate parameters:
fit.fkf <- optim(c(HHt = var(y, na.rm = TRUE) * .5,
                 GGt = var(y, na.rm = TRUE) * .5),
               fn = function(par, ...)
               -fkf(HHt = matrix(par[1]), GGt = matrix(par[2]), ...) $logLik,
               yt = rbind(y), a0 = a0, P0 = P0, dt = dt, ct = ct,
               Zt = Zt, Tt = Tt, check.input = FALSE)

## Filter Nile data with estimated parameters:
fkf.obj <- fkf(a0, P0, dt, ct, Tt, Zt, HHt = matrix(fit.fkf$par[1]),
              GGt = matrix(fit.fkf$par[2]), yt = rbind(y))

## Plot the width together with fitted local levels:
plot(y, main = "Treering data")
lines(ts(fkf.obj$att[1, ], start = start(y), frequency = frequency(y)), col = "blue")
legend("top", c("Treering data", "Local level"), col = c("black", "blue"), lty = 1)

## Check the residuals for normality:
plot(fkf.obj, type = "resid.qq")

## Test for autocorrelation:
plot(fkf.obj, type = "acf", na.action = na.pass)
```

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